Engineering Notes

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Theoretical Considerations in the Aerodynamic Effectiveness of Winglets

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Introduction

SINCE Whitcomb[1] presented the concept of "winglets" in the mid-1970's, a large number of studies concerning the effectiveness of winglets have been performed. As the result of these studies,[2,3] a winglet has been widely recognized as a more efficient means of improving cruise drag performance than a simple spanwise tip extension with the same structural weight penalty. Nevertheless, it is noteworthy that most of the practical applications of winglets have been designed on the existing wings and none of the major new aircrafts have incorporated winglets. Aircraft designers have not had sure convictions that such a nonplanar wing as a wing/winglet combination is more efficient than a conventional planar wing.

The objective of the present Note is to identify and explain the relative advantages of planar and nonplanar wings using simple theoretical analyses. As has been shown in the studies on this subject, it is necessary to take account of the structural constraint while comparing the aerodynamic efficiency. In this study, the wing-root bending moment was used as an index of the structural weight of a wing. In addition, the effect of the parasite drag has been included in the tradeoff study using an empirical method. The penalty in the parasite drag was found to be crucial in discussing the relative advantages of a winglet and a tip extension, although most of the previous theoretical studies have not taken this into account.

Minimum Induced Drag Comparison

First, the minimum induced drag for a nonplanar wing has been compared with that of a planar wing. According to Munk's theory,[4] the induced drag of any nonplanar wing in potential flow can be evaluated in the Trefftz plane. Reference 5 presents a simple numerical optimization method to calculate the minimum induced drag and the corresponding load distribution utilizing the simple vortex-lattice representation of Trefftz-plane wake and Lagrange multiplier technique. Such a method, originally developed by Lamar,[6] is applicable to arbitrary nonplanar configurations and has the capability of dealing with the root-bending-moment constraint. In this study, the following quantities were held constant for both planar and nonplanar wings when comparing the minimum induced drag value: 1) the total lift L, 2) the wing-root bending moment B, and 3) the perimeter length b in the Trefftz plane (not the projected span). For a prescribed value of design root bending moment and wing configuration, the minimum induced drag D_i can be evaluated by the numerical constrained optimization method. To generate a set of design charts, the value of root bending moment and wing configuration was varied parametrically.

One of the results obtained for wing/winglet combinations is shown in Fig. 1. The values of design root bending moment B and calculated minimum induced drag D_i are normalized using those for a planar wing with elliptic loading (denoted by the subscript elpl), which are constants since L and b are fixed in this analysis. The locus shown by the broken line corresponds to the minimum induced drag values without root-bending-moment constraints, which is in good agreement with those obtained from Munk's criterion on the minimum induced drag.[7] As is evident from Fig. 1, the minimum induced drag of a planar wing is smaller than that of a nonplanar wing of equal perimeter length except that the minimum induced drag becomes insensitive to tip shape variations for relatively small values of design root bending moment. This shows that from the viewpoint of the induced efficiency, a planar wing is superior to a nonplanar wing, even if the effect of the root bending moment is considered. A nonplanar wing tip is structurally beneficial in the sense that it has a very short moment arm compared with a planar wing. Nevertheless, Fig. 1 indicates that greater gain in the induced efficiency can be obtained by extending the wingspan and simultaneously shifting the load inboard such that the root bending moment is not increased. This conclusion is valid not only for wing/winglet combinations but also for more complex nonplanar configurations such as gull and curved wings.[5] Similar results have been obtained in Refs. 8 and 9.

Discussion—Importance of Parasite Drag Consideration

Although the above conclusion from the optimization problem is relevant only to the case of designing totally new wings, it apparently seems to contradict the results of the retrofit applications of winglets which show a decisive advantage for a winglet relative to a tip extension. However, it should be noted that the effect of the parasite drag on cruise drag performance has not been taken into account in the above analysis. According to Munk's stagger theorem,[4] all chordwise effects of a wing can be ignored in the evalutation of induced drag. Only the wake shape in the Trefftz plane and the load distribution can affect the value of induced drag. For example, the result of Fig. 1 for the induced efficiency is applicable to both a winglet and an endplate. On the contrary, the parasite drag does depend on the real configuration of a wing, particularly on the chord distribution because the parasite drag is roughly proportional to the wetted area. It should be remembered that an endplate, in contrast to a winglet, has been known to be ineffective for the purpose of reducing drag at the cruise condition because the parasite drag of an endplate which has large chord totally offsets the gain in the induced efficiency. This indicates that the result of Fig. 1 should be interpreted as follows: As long as the values of the parasite drag as well as the design root bending moment are kept constant, it is possible to design a planar wing so that it has better cruise drag performance than any nonplanar wing. On the contrary, this is not the case if the difference in the parasite drag level is com-

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parable to that in the induced drag. Since the chord length of winglet is smaller than that of a simple tip extension, it is found that the parasite drag consideration is indispensable to explain the relative advantages of a winglet and a spanwise tip extension.

To illustrate this, a numerical parametric study has been performed on a simplified wing/winglet system using a vortex-lattice program developed by Margason and Lamar.[10] The rectangular untwisted wing with an aspect ratio of 8.0 was selected as a baseline. The span length of an added tip surface was varied from 5 to 25% of the baseline wing semispan, and the dihedral angle ϕ from 0 to 90 deg. The chord length of an added surface was assumed to be equal to (case a) or narrower than (case b) the baseline wing. The wetted area of a surface in case b is half that in (case a) having the same span length. A simple spanwise tip extension ($\phi = 0$ deg) and a simple endplate ($\phi = 90$ deg) are included in case a. Case b represents winglets. For each modification, the values of induced drag and wing-root bending moment were calculated with a cruise lift coefficient of $C_L = 0.4$. All coefficients were defined based on the baseline wing geometry.

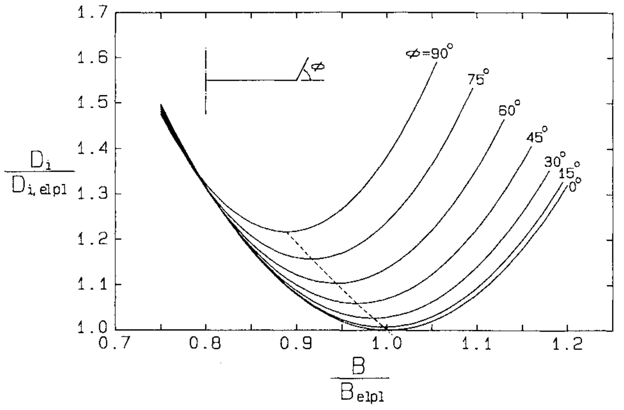

Fig. 1 Minimum induced drag as a function of root bending moment and winglet dihedral angle.

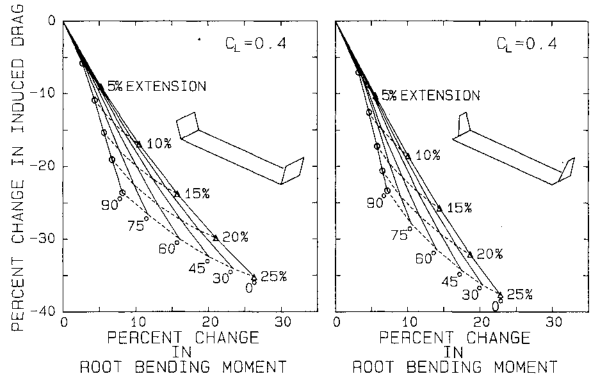

Fig. 2 Induced drag vs root bending moment trade for wing-tip modifications.

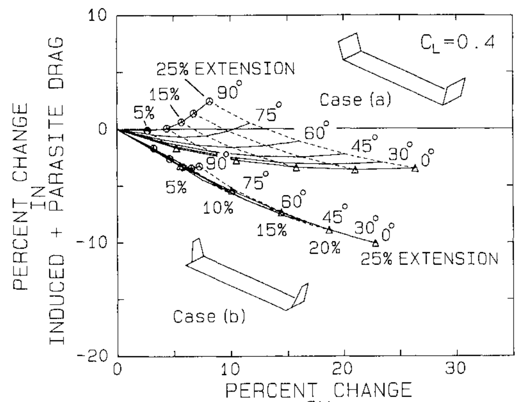

Fig. 3 Induced drag plus parasite drag vs root bending moment trade for wing-tip modifications.

Figure 2 shows the percent changes in the induced drag and the root bending moment from the values for the baseline wing alone. It can be observed that the effect of a winglet on reducing the induced drag is identical to that of an endplate. This result agrees with Munk's theory. Furthermore, it is shown in Fig. 2 that for the same increase in the root bending moment, highly nonplanar tip modifications ($\phi = 90$ deg, shown by circle) are about twice as effective in reducing the induced drag as planar ones (shown by triangle). Note that this results from the fact that a nonplanar tip surface can have larger span than a planar one with the same penalty in the root bending moment since the bending moment arm of a nonplanar tip surface is very small compared with that of a planar wing. However, it also should be noted that the parasite drag of a surface having the same chord length will increase with its span length. This suggests that a nonplanar tip device has a greater penalty in the parasite drag than a planar device with the same structural penalty. As discussed previously, it is not possible to compare the aerodynamic effectiveness of planar and nonplanar tip devices without considering the associated penalty in parasite drag.

Consequently, the problem has been re-examined by taking the parasite drag penalty into account. For simplicity, Hoerner's[11] empirical formula, based on the flat-plate surface friction, has been used to obtain profile drag at each span station, which was integrated along the span to give the total parasite drag of a wing. In calculating the profile drag, it was assumed that the value of Reynolds number was 10 million based on the baseline wing chord and both the wing and added surfaces had a constant thickness ratio of $t/c = 0.12$. Figure 3 indicates the results including the parasite drag penalty. It is shown that an endplate ($\phi = 90$ deg of case a) is not effective in reducing the overall drag (induced drag plus parasite drag) at cruise lift coefficient, and in contrast to an endplate, a simple spanwise tip extension ($\phi = 0$ deg of case a) as well as winglets (case b) still can produce considerable reduction in cruise drag. Further, a comparison of cases a and b shows that winglets are approximately twice as effective as a simple spanwise tip extension. These results agree well with the aircraft design experiences.

In addition, it should be emphasized that the results of Fig. 3 are reasonably consistent with the conclusion of the Trefftz-plane analysis on the point that a planar wing is superior to a nonplanar wing as long as the parasite drag consideration is taken. Figure 3 indicates that a winglet of 0 deg dihedral can provide a slightly greater improvement in cruise drag performance than a high-dihedral winglet. Thus, it is found that the primary reason a winglet is more effective at cruise lift coefficient than a spanwise tip extension is not that a winglet is nonplanar, but that a winglet has such a narrow chord that the relative penalty in the parasite drag is small compared with that for a spanwise tip extension having large chord. This suggests that a planar winglet of short span will be promising as an alternative tip device to a highly nonplanar winglet of large span, although its effectiveness, of course, may depend on the geometry of an unmodified wing.

Conclusions

To clarify the relative effectiveness between a winglet and a spanwise tip extension, simple theoretical analyses have been performed. The results obtained can be summarized as follows.

1) To determine the relative effectiveness between a winglet and a spanwise tip extension, the tradeoff between the induced

efficiency and the root bending moment is insufficient. The consideration in the parasite drag penalty associated with the tip modifications is indispensable.

2) As long as the parasite drag and root bending moment are kept constant, it is possible to design a more efficient planar wing than any nonplanar wing having equal perimeter length.

3) The primary reason that a winglet is more effective than a spanwise tip extension in reducing cruise drag is not that a winglet is nonplanar, but that a winglet has such a narrow chord that the relative penalty in the parasite drag is small compared with that for a spanwise tip extension having large chord.

4) A planar winglet of short span is a promising alternative to a highly nonplanar winglet of large span, although its effectiveness may depend on the geometry of an unmodified baseline wing.

Acknowledgments

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Dynamic Overshoot of the Static Stall Angle

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THE extensive set of experimental results for an airfoil in pitch-up motion presented by Daley and Jumper[1] indeed, as they state, ". . . represents a viable data set against which theoretical predictions of dynamic separation may be compared." If one neglects any existing differences between the measured angle of attack for flow separation at 25% chord[1] and the dynamic stall angle, Ref. 2 provides the following analytic prediction of the dynamic overshoot $\Delta\alpha_{sep}$ of the static stall angle.

$$\Delta\alpha_{sep} = \Delta a_w + \Delta\alpha_s \tag{1}$$

where $\Delta\alpha_w$ is given by Eq. (1) of Ref. 2

$$\Delta\alpha_w = 1.5 \;\; c\dot{\alpha}/U_\infty = 3.0\dot{\alpha}_{ND} \tag{2}$$

and $\Delta\alpha_s$ by Eq. (6) of Ref. 2

$$\Delta\alpha_s = K_a \;\; c\dot{\alpha}/U_\infty = 2K_a\dot{\alpha}_{ND}$$

$$K_a = K_{a1} + K_{a2}\xi_{CG} \tag{3}$$

As $\Delta\alpha_s$ represents the overshoot of the static stall angle due to the beneficial effects on the boundary layer characteristics generated by the airfoil pitch-up motion, it reaches an asymptotic limit for large $c\dot{\alpha}/U_\infty$. This so called "infinite Reynolds number limit"[2] is reached when the pitch-up rate exceeds

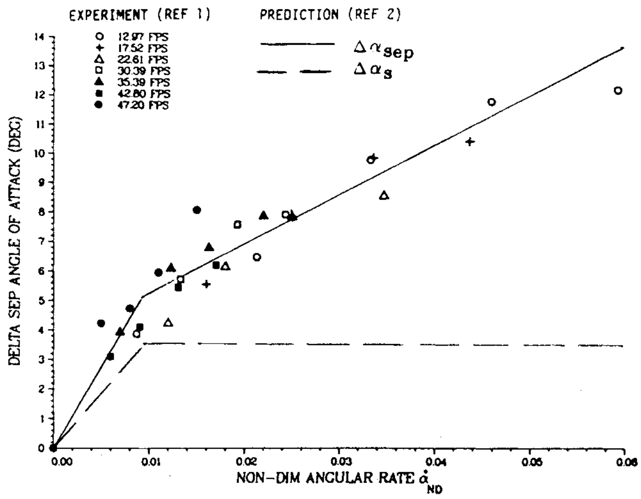

Fig. 1 Dynamic overshoot of static flow separation angle of attack at moderate pitch-up rates.

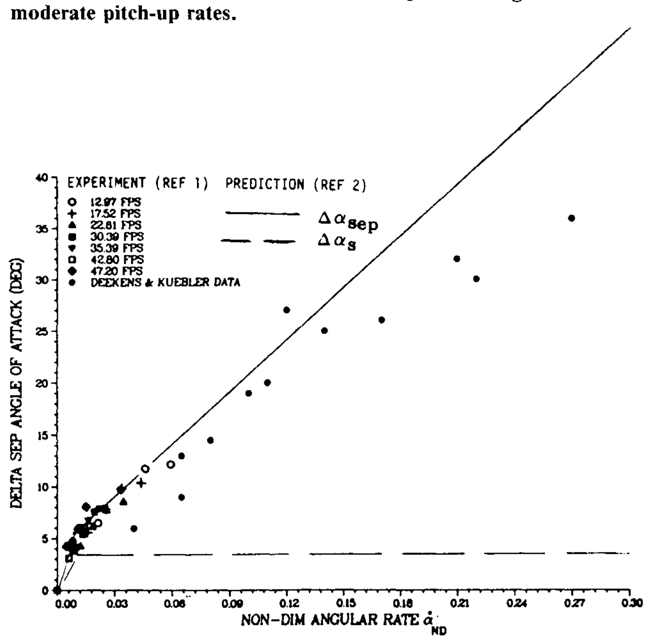

Fig. 2 Dynamic overshoot of static flow separation angle of attack at high pitch-up rates.

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